CFD Model of wind turbine wake in atmospheric turbulence using body forces

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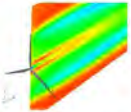
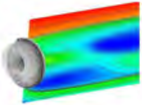
Work context

- The aim of this work is to develop a wind farm wake model in Computational Fluid Dynamics (CFD), to support engineering wind farm wake model development (and test their assumptions).
- The turbine is modeled as a porous disc of forces (Actuator Disc).
  - The forces and power are found using C_T and C_P curves, similarly to the engineering models
- Steady state simulations
  - (Much) quicker than unsteady computations.
  - Also a good a way to validate qualitatively the assumptions made in engineering models.
  - Appropriate to comparison with 10-minute average wake data ?
- RANS Turbulence: k-ϵ
  - Risø has a long experience on atmospheric cases
  - <u>But:</u> The wake development does not give satisfying results.... $\Longleftarrow$



Full rotor models
1-CPU time: months



Actuator disc models
1-CPU time: hours

Engineering models
1-CPU time: seconds

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The model does not compare well with measurements

- Nibe turbine wake measurement (Taylor 1990[1])
- $C_T = 0.89$, $z_0 = 0.1\text{m}$, $D = 40\text{m}$
- The wind has been measured at 2.5D, 4D, 6D and 7.5D behind the turbine.

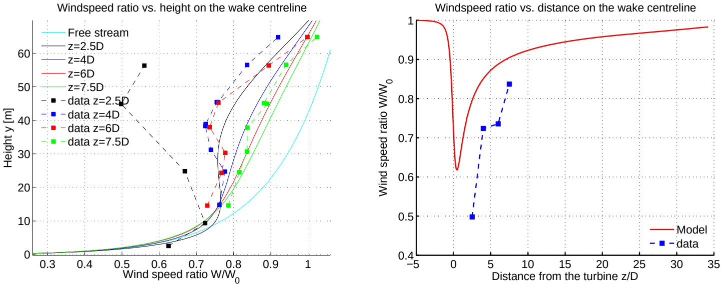

Figure: Comparison of the standard k-ϵ model with the Nibe measurements

[1] Taylor G.J., 1990, Wake Measurements on the Nibe Wind Turbines in Denmark, National Power, ETSU WN 5020.

The model does not compare well with LES simulations

- LES turbulence is separated in two parts: the large eddies are simulated, and the small eddies are modeled (eddy size $<$ grid size).
- Comparison of the mean flow features between a RANS and a time-averaged LES computation (3000 iterations)

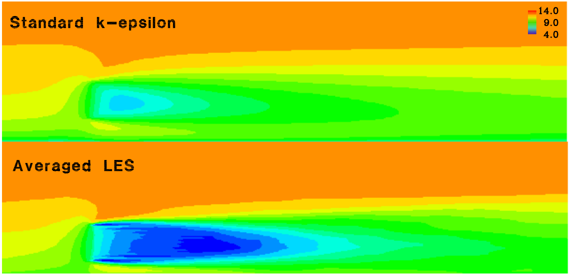

Figure: Comparison of the standard k-ϵ model with the averaged LES model. Side view.

Analytical presentation of the problem

- RANS Navier stoke equation:
  $\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}\left[(\mu + \mu_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + \frac{\partial P}{\partial x_i} = S_U$
- The turbulence models are used to determine the eddy viscosity μ_t.
- In order to be in agreement with the law of the walls, and obtaining a logarithmic profile, the eddy viscosity must be linearly dependent with the height $\mu_t = \rho \kappa u_\tau y$.
- We would like to know what is the effect of an atmospheric eddy viscosity on the wake development
- Let's take a primitive turbulence model where we keep the eddy viscosity constant inside our domain.
- It's equivalent to changing the molecular viscosity.

Wake behavior under different effective Reynolds numbers

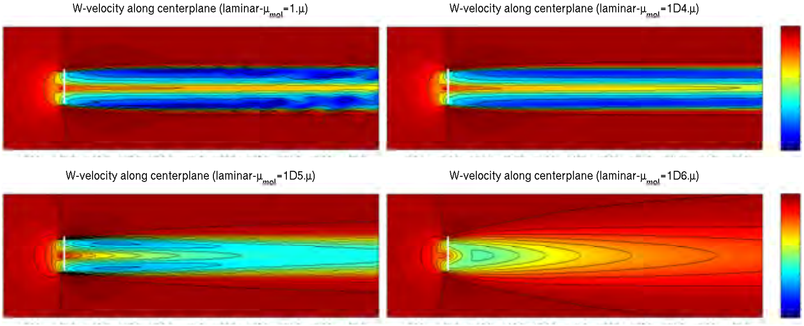

Figure: Laminar: W-velocity for different molecular viscosities (Re=1D8, 1D4, 1D3, 1D2)

Study of a typical k-ϵ computation

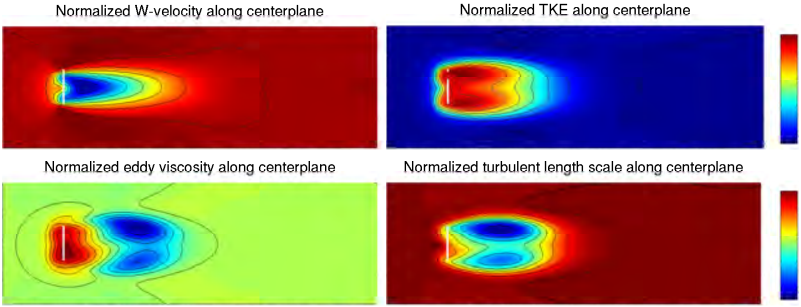

Figure: Wind turbine seen from the top: W-velocity, TKE, viscosity and turbulent length scale normalized by the atmospheric values

Team Montreal

- El Kasmi & Masson[1] propose to use Chen[2] model to modify the k-ϵ model.
- It adds a term in the dissipation equation: $C_{\epsilon 4} P_t^2 / \rho k$
- It's basically tuning the turbulence kinetic energy dissipation equation so that the dissipation reacts faster to the change of turbulence kinetic energy production.
- They propose to only apply this term in the surrounding of the wind turbine, because it's an area of turbulence imbalance.
- They also have a turbulence source term accounting for the tip vortices, estimated using a BEM.
- It basically gives 2 free parameters ($C_{\epsilon 4}$, and the area size) to control the dissipation of turbulence.

[1] El Kasmi A., Masson C., 2008. / J. Wind Eng. Ind. Aerodyn. 96 (2008) 103122.
[2] Chen, Y.S., Kim, S.W., 1987. / NASA Contractor Report, NASA CR-179204.

Team DTU

- DTU is modelling the boundary layer using body forces instead of a turbulence model.
- This way they can have a very low eddy viscosity (so very high effective Reynolds nb).
- It seems to be a good way to study the close wake area.
- The problem is that there is no guaranty that the far wake recovery will be realistic as there is no boundary layer turbulence.

Canopy modelling in CFD using body forces

- +30 years of experience
- Hot topic in wind energy
- ex: Sogachev[1], Sanz[2] both propose to add some source/sink terms in the k and ϵ equations
- From Sanz[2]:
  - momentum-equation: $S_U = \frac{C_X}{2} U^2$
  - k-equation: $S_k = \frac{C_X}{2} (\beta_p U^3 - \beta_d U k)$
  - ϵ-equation: $S_\epsilon = \frac{C_X}{2} (C_{\epsilon 4} \beta_p \frac{\epsilon}{k} U^3 - C_{\epsilon 5} \beta_d U \epsilon)$
  - $\frac{C_X}{2} \beta_p U^3$ is the turbulence wake production rate, where β_p is the fraction of mean airflow kinetic energy lost by drag that is converted into k.
  - $\frac{C_X}{2} \beta_d U k$ is the turbulence sink accounting for the shortcircuiting of turbulence cascade, where β_d has *"no clear physical basis"*.

[1] Sogachev A. 2009, A Note on Two-Equation Closure Modelling of Canopy, Boundary-Layer Meteo 130:423-435
[2] Sanz C. 2003, A Note on k-ϵ Modelling of Vegetation Canopy Air-flows, Boundary-Layer Meteo 108:191-197

β_p

- β_p is "*the fraction of mean airflow kinetic energy lost by drag that is converted into k*". So it should be proportional to the difference between **the extracted power** and **the power lost by the mean airflow**.
- From basic actuator disc theory:
  - Power lost by the mean airflow: $P_{lost} = \frac{1}{2}\rho A C_P U_\infty^3$, with $C_P = 4a(1-a)^2$
  - Wind Turbine Thrust: $T = \frac{1}{2}\rho A C_T U_\infty^2$, with $C_T = 4a(1-a)$
  - Induced velocity factor: $a = \frac{1}{2}(1 - \sqrt{1 - C_T})$
  - Velocity at the disc $U_D = \frac{U_\infty}{1-a}$
- We have the real thrust and power from the wind turbine. So the power lost by the mean airflow can be expressed using the real thrust coefficient $P(C_{T,real})$.
  - $P_{lost} = \frac{1}{2}\frac{4a}{1-a}\rho A U_D^3$
  - $P_{turb} = P_{lost} - P_{extracted}$
  - $P_{turb} = \frac{2a}{1-a}\rho A \left(1 - \frac{C_{P,real}}{4a(1-a)^2}\right) U_D^3 \qquad \approx \qquad \frac{C_X}{2}\beta_p U^3$

β_d

- To determine β_d, we go back to the RANS derivation.
- RANS Navier stoke equation:
  $\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}\left[(\mu + \mu_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + \frac{\partial P}{\partial x_i} = S_U$
- Let's take a closer look at S_U:
  - $S_U = \frac{2a}{1-a}\rho A U_D^2$
  - Reynolds averaging: $\overline{(U+u')(U+u')} = UU + 2\not{\overline{u'}}U + \overline{u'u'}$
  - how big is $\overline{u'u'}$?: if $TI = \sqrt{\overline{u'u'}}/U < 0.1 \Longrightarrow \overline{u'u'} < 0.01U^2$
- What happens to S_U when derivating the k-equation?:
  - k is found by rearranging $\overline{u'_j NS(U_i + u'_i)} = 0$
  - $\overline{u' S_U(U+u')} = \frac{2a}{1-a}\rho A(\not{\overline{u'}}UU + \overline{u'u'}U + \overline{u'u'u'})$
  - $\overline{u'u'}U \propto \frac{8}{9}kU \qquad (k = \frac{1}{2}\overline{u'_i u'_i})$ and $(\overline{u'^2} : \overline{v'^2} : \overline{v'^2} \equiv 4:2:3)$
  - $\overline{u'u'u'} \propto -\frac{\nu_T}{\sigma_k}\frac{\partial k}{\partial x_j} \qquad$ (analogy to molecular process, Wilcox[1])

[1] Wilcox, D.C. 2006, Turbulence Modeling for CFD

Summary

- Momentum-equation: $S_U = \frac{2a}{1-a}\rho A U_D^2$
- k-equation: $S_k = \frac{2a}{1-a}\rho A \left[\left(1 - \frac{C_{P,real}}{4a(1-a)^2}\right) U_D^3 - \frac{8}{9} k U_D\right]$
- ϵ-equation: $S_\epsilon = \frac{2a}{1-a}\rho A \left[C_{\epsilon 4}\left(1 - \frac{C_{P,real}}{4a(1-a)^2}\right) U_D^3 \frac{\epsilon}{k} - C_{\epsilon 5}\frac{8}{9}\epsilon U_D\right]$

Comparison of the canopy model with the Nibe measurements

- Nibe turbine wake measurement (Taylor 1990[1])
- $C_T = 0.89$, $z_0 = 0.1\text{m}$, $D = 40\text{m}$
- The wind has been measured at 2.5D, 4D, 6D and 7.5D behind the turbine.
- $C_{\epsilon 4} = 0.25$ and $C_{\epsilon 5} = 1.0$

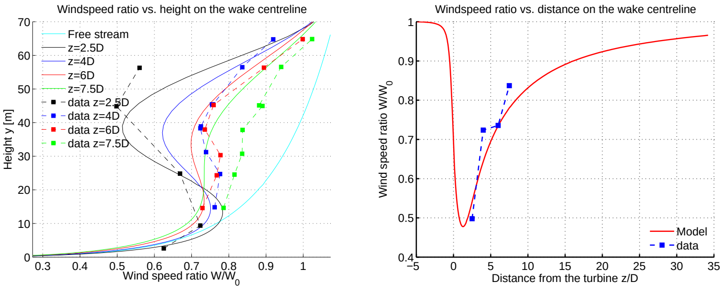

Figure: Comparison of the canopy k-ϵ model with the Nibe measurements

[1] Taylor G.J., 1990, Wake Measurements on the Nibe Wind Turbines in Denmark, National Power, ETSU WN 5020.

Canopy k-ϵ model

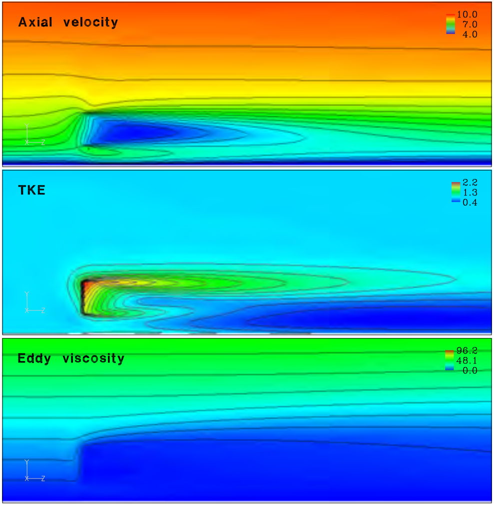

Figure: Wind turbine seen from the side: axial velocity, TKE, Eddy viscosity

Conclusion

- Proposing to apply a canopy model for modelling wind turbine wake turbulence.
- Derivation of the model variables from basic actuator disc theory and RANS theory.
- The model seems to act correctly.
- The comparison with some measurements are encouraging
- Paper to be presented at EWEC 2009.

Future work

- Full parameter study.
- Comparison with with more single wake measurements.
- Comparison with offshore wind farm measurements.

Question for the public

Question: How do you model the turbulence in atmospheric boundary layer?